Assignment 3 Solution

Exercise 1:

a) All of the professors are philosophy professors and understand logic
b) There exist a professor that is a philosophy professor and understand logic
c) All philosophy professors understand logic
d) There exist a philosophy professor that understand logic

Exercise 2:

a) $\exists x(I(x) \land S(x) \land N(x))$
b) $\neg\exists x(I(x) \land S(x) \land N(x)) \equiv \forall x(\neg I(x) \lor \neg S(x) \lor \neg N(x))$
c) $\forall x(I(x) \lor S(x) \lor N(x))$
d) $\exists x(I(x) \land \neg S(x) \land N(x))$
e) $\neg\exists x(I(x) \lor S(x) \lor N(x)) \equiv \forall x(\neg I(x) \land \neg S(x) \land \neg N(x))$

Exercise 3:

a) P(2): 3>3 => False
b) P(3): 7>5 => True
c) P(0): -5>-1 => False
d) $\exists x$ P(x): True (for x=3 P(3) is true)
e) $\exists x \neg P(x)$: True (for x=0 p(0) is false)
f) $\forall x$ P(x): False (for x=2 p(2) is false)
g) $\forall x \neg P(x)$: False (for x=3 P(3) is true)

Exercise 4:

a) $P(-4) \lor P(-2) \lor P(-1) \lor P(1) \lor P(2) \lor P(4)$
b) $P(-4) \land P(-2) \land P(-1) \land P(1) \land P(2) \land P(4)$
c) $((-4 \neq 3) \text{ --> } P(-4)) \land ((-2 \neq 3) \text{ --> } P(-2)) \land ((-1 \neq 3) \text{ --> } P(-1)) \land ((4 \neq 3) \text{ --> } P(4)) \land ((2 \neq 3) \text{ --> } P(2)) \land ((1 \neq 3) \text{ --> } P(1))$
d) $(-4 \geq 1 \land P(-4)) \lor (-2 \geq 1 \land P(-2)) \lor (-1 \geq 1 \land P(-1)) \lor (4 \geq 1 \land P(4)) \lor (2 \geq 1 \land P(2)) \lor (1 \geq 1 \land P(1))$

e) (¬P(-4) ∨ ¬P(-2) ∨ ¬P(-1) ∨ ¬P(1) ∨ ¬P(2) ∨ ¬P(4)) ∧ [(-4<0 -->P(-4) ∧ (-2<0 --> P(-2) ∧ (-1<0 --> P(-1) ∧ (4<0 -->P(4) ∧ (2<0 --> P(2) ∧ (1<0 --> P(1)]

Exercise 5:

Let C(x) be x is in my classroom
The domain of x is all students in my class
The domain of y is all people

a) Let D(x) be x can drive
   1. ∃x(¬D(x))
   2. ∃y(C(y) ∧ ¬D(y))

b) Let N(x) be x has a notebook
   1. ∀x(N(x))
   2. ∀y(C(y) → N(y))

c) Let R(x) be x is repeating this course
   1. ∃x(R(x))
   2. ∃y(C(y) ∧ R(y))

d) Let F(x) be x can solve Fermat's last theorem
   1. ∀x(F(x))
   2. ∀y(C(y) → F(y))

e) Let W(x) be x wants to be rich
   1. ∃x(¬W(x))
   2. ∃y(C(y) ∧ ¬W(y))

Exercise 6:

a) There exist a CMPS professor that taught a CMPS course
b) There exists a CMPS professor that taught all CMPS courses
c) All CMPS professors taught a CMPS course

d) There exists a CMPS course that has been taught by all CMPS professors
e) All CMPS courses has been taught by CMPS professors
f) All CMPS professors taught all CMPS courses

Exercise 7:

a) Tarek Haddad is enrolled in CMPS 211
b) There exists a student in my school that is enrolled in Bio 210
c) There exists a class in my school that David Cook is enrolled in
d) There exists a student that is enrolled in Math 201 and in CMPS 211
e) There exists two different students such that if the first one is enrolled in any course, then the second one is enrolled in the same course
f) There exists two different students that are both enrolled in the same class or none of them is enrolled in it

Exercise 8:

a) $\forall x \forall y \neg Q(x, y)$
b) $\exists x \exists y Q(x, y)$
c) $\exists x (Q(x, AMC) \wedge Q(x, IEEE))$
d) $\forall y \exists x Q(x, y)$
e) $\exists x_1 \exists x_2 (Q(x_1, IEEE) \wedge Q(x_2, IEEE))$

Exercise 9:

The domain of x is “the students in this class” for [a-g]

a) Let S(x, y) be x can speak y
   The domain of y is “languages”
   $\exists x S(x, Aramaic)$

b) Let E(x, y) be x has eaten y
   The domain of y is “fruits”
   $\exists x (E(x, pears) \wedge \neg E(x, peaches))$

c) Let I(x, y) be x plays y
The domain of y is “instruments”
$\forall x \exists y\ I(x, y)$

d) Let L(x, y) be x learned y
The domain of y is “programing languages”
$\forall x \exists y\ L(x, y)$

e) Let G(x, y) be x can speak y
The domain of y is “towns”
$\exists x_1 \exists x_2 \exists y\ (G(x_1, y) \wedge G(x_2, y))$

f) Let B(x, y, z) be x has eaten y in z
The domain of y is “dishes offered”
The domain of z is “restaurants on Bliss Street”
$\exists x \forall y \exists z\ B(x, y, z)$

g) Let P(x, y, z) be x has played with y in game z
The domain of x and y is “Lebanese soccer players”
The domain of z is the “games that has been played”
$\forall x \forall y \exists z\ P(x, y, z)$

Exercise 10:

The domain of x is “the students in this class”

a) Let S(x, y) be x failed y
The domain of y is “courses”
$\exists x \forall y \neg S(x, y)$
Negation: $\forall x \exists y\ S(x, y)$
All of the students in this class have failed a course

b) Let L(x, y) be x likes y
The domain of y is “majors”

∀x L(x, CMPS)

Negation: ∃x ¬L(x, CMPS)

There exist some students in this class who hate CMPS

c) Let M(x, y) be x has taken y

The domain of y is "mathematics courses offered in this school"

∃x ∀y M(x, y)

Negation: ∀x ∃y ¬M(x, y)

All of the students in this class have not taken mathematics courses offered by this school

d) Let B(x, y, z) be x has been in y of z

The domain of y is "rooms of buildings"

The domain of z is "buildings in Beirut"

∃x ∃y ∀z B(x, y, z)

Negation: ∀x ∀y ∃z ¬B(x, y, z)

All the students in this class have not been in every room in at least one building in Beirut